

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 15 Jan. 1995		3. REPORT TYPE AND DATES COVERED Final Technical Report
4. TITLE AND SUBTITLE Final Technical Report Robust Control Methods AFOSR Grant F49620-92-J-0014			5. FUNDING NUMBERS F49620-92-J-0014	
6. AUTHOR(S) Michael G. Safonov				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) University of Southern California Department of Electrical Engineering Systems 3740 McClintock EEB 308 MC2563 Los Angeles, CA 90089-2563			8. PERFORMING ORGANIZATION REPORT NUMBER AFOSR-TR-95-0185	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) Air Force Office of Scientific Research/NM AFOSR/PKD 110 Duncan Avenue, Suite B115 Bolling AFB, DC 20332-0001			10. SPONSORING/MONITORING AGENCY REPORT NUMBER <div style="border: 2px solid black; padding: 5px; text-align: center;"> DTIC SELECTED JUN 16 1995 F </div>	
11. SUPPLEMENTARY NOTES Reporting period: November 1, 1991 to January 15, 1995				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unlimited/Unclassified			12b. DISTRIBUTION CODE <div style="border: 1px solid black; padding: 5px; text-align: center;"> IRS RECEIVED FROM DISTRICT 05 09 95 </div>	
13. ABSTRACT (Maximum 200 words) This is the final report for research supported under AFOSR Grant F49620-92-J-0014 during the period November 1, 1991 through January 15, 1995. During this period research effort was broadly focused on developing the theory of extending class of solvable robust control problems and on developing a theory to accommodate the issues that arise in going from experimental data to robust control designs. Robust control concerns the problem of engineering control systems capable of robustly maintaining performance to within prescribed tolerances in the face of large-but-bounded modeling uncertainties and nonlinearities. A significant breakthrough achieved by the research is a new and remarkably simple <i>Bilinear Matrix Inequality (BMI)</i> embedding of the robust control problem which distills the essential mathematical features of the full spectrum of robust control problems. Besides being simple and mathematically elegant, the BMI significantly expands the class controller design constraints that can be accommodated to include reduced order μ/K_m control, decentralized control, multimodel control, gain-scheduling, mixed H^2/H^∞ control and so forth. A second breakthrough has been the introduction of a new "unfalsified control concept" providing a remarkably lucid mathematical characterization of the processes of learning and adaptation in robust control design; this theory is expected to lead to much improved techniques for reducing experimental data to practical and reliable control designs for a variety of advanced aerospace engineering applications where robust performance is prerequisite, e.g., aircraft stability augmentation systems, highly maneuverable aircraft design, missile guidance systems, and precision pointing and tracking systems.				
14. SUBJECT TERMS			15. NUMBER OF PAGES 2 16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT Unlimited	

Final Report:
ROBUST CONTROL METHODS
 AFOSR Grant F49620-92-J-0014

November 1, 1991 – January 15, 1995

Michael G. Safonov
 Department of Electrical Engineering
 University of Southern California
 Los Angeles, CA 90089-2563

January 15, 1995

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1 Research Objective

Research efforts supported by AFOSR Grant F49620-92-J-0014 have been a continuation of a broader plan supported by previous AFOSR grants to develop a cohesive, complete theory for the design of systems for precision control of uncertain, highly nonlinear systems including, but not limited to, high performance military aircraft flight control, laser-based tracking and targeting sensors, missile autopilots, and so forth. While such applications form the context of the research, the focus has been on developing the mathematical concepts and theory needed to formulate, analyze and solve such problems in an engineering setting.

2 Accomplishments

Fifty-four publications supported under AFOSR Grant F49620-92-J-0014 have either appeared, been submitted or are currently pending publication [1]–[54]. Areas of significant progress represented by these AFOSR supported publications include the following:

- Linear System Theory [3, 4]
- Modeling Accuracy Needs in Identification for Control [24, 10]
- Model Reduction and Identification for Control [7, 9]
- Real Multivariable Stability Margin (MSM) Analysis [8, 5, 11, 10, 6, 24, 30, 52, 10, 11, 31, 40]
- Theory for Reliable Numerical Computation of H_∞ Controllers [23, 4, 17, 27, 28, 28, 33, 34, 20]
- How to Use H_∞ Control Theory in Design [2, 30, 25]
- Beyond H_∞ Control [26, 18, 52]
- Bilinear Matrix Inequality (BMI) control synthesis [29, 37, 38, 41, 44, 39]
- Unfalsified-Control/Set-Theoretic-Adaptor-Control Systems [20, 21, 36, 32, 40, 22, 54]
- H_∞ Aerospace Control Design [1, 2, 42]

Most of the theoretical developments embodied in the above listed recent AFOSR publications have been, or will soon be, implemented in software. Other concepts developed with AFOSR support played a central role in a supermaneuverable fighter aircraft control design study [1] — and, evidently, in the Air Force Wright Laboratory follow-on study [65]. The generalized Popov multiplier robustness analysis concepts developed in [53, 31, 19, 29, 40] have led directly to improved approaches for the design of active vibration damping systems for flexible space structures [42]. The effective and rapid transition from theory to practice has been facilitated by my on-going non-AFOSR-supported involvement with Dr. R. Y. Chiang in creating, and periodically upgrading, the MATLAB ROBUST CONTROL TOOLBOX, a robust control design software product published by The MathWorks and in use on more than 1000 government and industrial computer systems [64]. Further details of several of the most significant achievements are elaborated below.

Model Reduction and Identification for Control

Multiplicative error bounds developed in [9] had important practical implication for generating models suitable for use in H_∞ control design since robustness is assured if multiplicative model error is less than one inside a control system's bandwidth. Going beyond model reduction to the identification of models from data; the Balanced Stochastic Truncation model reduction method was developed into an engineering tool that can take raw autocorrelation data and generate a low order linear time-invariant stochastic realization with a prespecified relative-error [7]. In view of my earlier "robustness criterion" for modeling (which, loosely speaking, says simply that a relative-error smaller than one means a model is adequate for control system design), the potential of this system identification technique for use in robust control design is enormous. Unfortunately, when the autocorrelation data comes from (possibly noisy) input-output measurements from a plant to be controlled, the relative-error bound is not on the plant itself, but rather on a realization of a phase-blind stochastic realization of the measurement data, so that additional work will be required to turn this into a practical tool. The paper [7] includes some preliminary ideas which may, with substantial improvement, lead me to the result which I seek — a method to compute multiplicative-error bounded models directly from input-output autocorrelation data.

Singular H_2 and H_∞ Control

Significant progress in advancing H_∞ control theory for "singular plants," i.e., those with zeros either on the $j\omega$ -axis or at ∞ . Such singular problems are actually far from rare. For example, the classical H_∞ sensitivity minimization problem introduced by Zames in 1981 results in an improper control law having unobservable poles at $\omega = \infty$ when his Nevanlinna-Pick interpolation approach is applied and the plant is strictly proper. In these situations the state-space H_∞ theory fails to produce any solution at all. A different — but related — problem occurs when one attempts to apply weighted mixed-sensitivity H_∞ control synthesis to a plant with a pole at $s = 0$; e.g., a system having proportional-integral feedback. Such situations are the bread and butter of control engineers and cannot be realistically ignored. Preliminary results concerning a solution to these previously unsolved singular H_∞ control problems are described in my papers with Copeland and with Goh [23, 4, 17, 27, 28, 28, 33, 34, 20]. As is shown in [27], these results can be further simplified so as to permit faster, more reliable computation of H_∞ control laws for singular plants. In the coming months, I will examine the remaining problem of finding a representation of the subset of proper, internally-stabilizing H_∞ control laws for singular plants.

Real K_m Analysis and Synthesis

Using a variant on the Popov multiplier technique from nonlinear stability theory, combined with the H_∞ synthesis theory via bilinear sector transform, I have developed the theoretical framework to significantly reduce the conservativeness with which uncertain real parameters are handled [31, 19, 29]. Our results eliminate the difficult and awkward "curve fitting" step associated with previous approaches to K_m -synthesis. They constitute a major theoretical breakthrough, making reliable, fully-automated K_m -synthesis theoretically possible for the first time.

Bilinear Matrix Inequality (BMI) Synthesis

As shown in papers [37, 44, 14], a broad spectrum of robust control problems, including multimodel, decentralized, and reduced-order μ/K_m -synthesis problems, can be reformulated as Bilinear Matrix Inequality (BMI) Feasibility Problems. The BMI is an extension of the Linear Matrix Inequality

(LMI) approach that has recently been found to be useful in formulating and solving a limited class of robust control problems, including state-feedback and full-order dynamical output feedback H_∞ control, μ/K_m analysis, simultaneous stabilization, gain-scheduling, and so forth (e.g., [66, 67, 68, 69, 70, 71, 72, 73, 74, 32]). In particular, the BMI formulation offers the advantage of *simultaneously* handling all the foregoing types of specifications *as well as additional specifications not amenable to the LMI framework such as constraints on controller structure (e.g., decentralized "block-diagonal" control) and on controller order*. The BMI formulation also sheds new insight into the properties and limitations of existing robust control algorithms such as the μ/K_m -synthesis, indicating that the classical DK -iteration may not even produce locally optimal solutions.

Mathematically, the BMI is defined as follows:

Definition 2.1 (BMI Feasibility Problem) *Given real Hermitian matrices $F_{i,j} = F_{i,j}^T \in \mathcal{R}^{m \times m}$, for $i \in \{1, \dots, n_x\}$, $j \in \{1, \dots, n_y\}$. Define the matrix-valued bilinear function $F : \mathcal{R}^{n_x} \times \mathcal{R}^{n_y} \rightarrow \mathcal{R}^{m \times m}$:*

$$F(x, y) \triangleq \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} x_i y_j F_{i,j} \quad (2.0)$$

*Find, if they exist, real vectors $x = [x_1, \dots, x_{n_x}]^T \in \mathcal{R}^{n_x}$ and $y = [y_1, \dots, y_{n_y}]^T \in \mathcal{R}^{n_y}$ such that $F(x, y)$ is positive definite. This is called the **bilinear matrix inequality feasibility problem**.*

The global solution of such BMI's would resolve many of the major limitations the existing μ/K_m -synthesis theory for robust control design.

For example, as shown in [37], BMI's provide a natural formulation for the problem of optimal reduced-order H_∞ control synthesis introduced by [75, 76]. The BMI formulation seems to us rather simpler than the nonlinearly coupled LMI's proposed in [72, 73]. Unlike standard LMI's, such nonlinearly coupled LMI's have so far defied attempts to develop globally convergent solution algorithms.

Likewise, while the controller structure constraints required in the synthesis of decentralized controllers have so far defied attempts to embed them in the LMI framework, these constraints are readily embedded within the BMI framework. Even more importantly, the BMI framework naturally handles the μ/K_m -synthesis with fixed-order generalized Popov multipliers [19, 29].

Recall that while each of the two problems of solving for an optimal Popov multiplier $M(s)$ with the controller $K(s)$ fixed, and then solving for an optimal H_∞ controller $K(s)$ with the multiplier $M(s)$ fixed, is convex, the μ/K_m problem is not *jointly* convex in the multiplier and the controller. The upshot is that no guarantees of convergence to globally optimal values of $M(s)$ and $K(s)$ are possible. Indeed, solutions may not even be locally optimal. But, with the aid of the generalized positive real lemma [77], we show in [37] that decentralized and reduced-order μ/K_m -synthesis control problems admit simple BMI formulations.

Our preliminary study [38] of the properties of the BMI feasibility problem indicates that it is possible to obtain local solutions which at least improve on existing alternating D - K synthesis techniques. However, the problem in general requires globally optimal solutions. In this regard, we find it encouraging that the global solution of a BMI has the simple interpretation that it is equivalent to finding the diameter of a collection of origin-centered ellipsoids in \mathcal{R}^N [44]; i.e., it is the diameter of a very simple, highly structured convex set. One of the chief goals of our future research will be to develop reliable general purpose BMI solution algorithms which fully exploit the underlying simplicity and structure of the BMI problem.

Unfalsified Control Theory

Inspired by the “unfalsified model” concepts used in model validation (e.g., [78, 79, 80, 81, 82, 83, 84]), but disappointed by their relative complexity and inherent conservativeness when used for control-oriented identification in conjunction with state-of-the-art robust control methods, a more direct “unfalsified control” approach was introduced by us in [22, 41, 45, 13] — see also [20, 21, 32, 36, 54].

Apparently new, our unfalsified control concept is a “model-free” approach to control. It works directly with input-output measurement data with the only model required being that of a parameterized class of candidate control laws. The central idea in our unfalsified control approach is that controller models can be “validated” against performance specifications directly from plant input-data without any need to identify or validate models of the plant itself. Furthermore, the computations required for direct “controller validation” are really no more difficult than those required for plant model validation of the type in [80, 79, 82, 84, 83, 85].

Thus, instead of attempting to enforce a somewhat artificial separation between modeling and control design, our unfalsified control concept dispenses with plant models and uncertainty models altogether, focusing instead directly on the controller model and the implications of the available plant data regarding its capability to meet performance specifications. It replaces the conventional indirect two-step approach of (a) finding unfalsified plant models and (b) designing robust controllers. Our concept takes one directly from plant input-output data to control designs without the necessity of plant or uncertainty modeling. This is possible since all needed information about the plant is already in the plant input-output data — and this information turns out to be sufficient to validate control laws.

The essence of the unfalsified control concept is depicted abstractly in Figure 1. The three axes represent the three (infinite dimensional) function spaces of which the signals r, y, u are members. The three signals r, y, u are, respectively, commands $r(t)$, plant output $y(t)$ and control signal $u(t)$. In this context, a plant is a collection of input-output signal pairs (u, y) . A control design specification is a constraint on the signal pairs (r, y) — i.e., a set, say \mathbf{T} , in which the pair (r, y) must lie. A control law, say K , is a constraint on the triple (r, y, u) , i.e., a subset of the set of triples (r, y, u) . In Figure 1 the plane $K(u, y, r) = 0$ represents a particular linear control law. The key observation is that one may test consistency of the control law $K(u, y, r) = 0$ with the specification \mathbf{T} and the past plant data (u, y) by checking that the image of the pair (u, y) under the constraint $K(u, y, r) = 0$ is a pair (r, y) in \mathbf{T} . Moreover, this controller consistency test may be performed even if the plant data (u, y) has been generated by another control law — or even if it has been generated open-loop with no control law at all. A control law K which fails to be consistent with the performance specification and the past plant input-output data is invalidated, i.e., *falsified*; those control laws which are not falsified are said to be *unfalsified*.

This simple idea is our *unfalsified control concept*. But, simple though it may be, it is a revolutionary concept. It makes no explicit use of plant models other than the data itself, so in this sense it is a “model-free” approach to control. Because it requires no unverifiable assumptions and works only with data and specifications, it provides a direct, nonconservative approach to control design, as illustrated by the example in [45, 13].

Current research aims to turn the unfalsified control concept into a practical theory for robust control design. The “ACC Benchmark” robust control design problem solved by us using unfalsified control techniques in [22, 54] establishes not only the conceptual feasibility of the unfalsified control approach, but also that it can actually lead to superior designs. Additionally, it appears that our unfalsified control concept will lead to a more scientific basis for the study of adaptive control and learning.

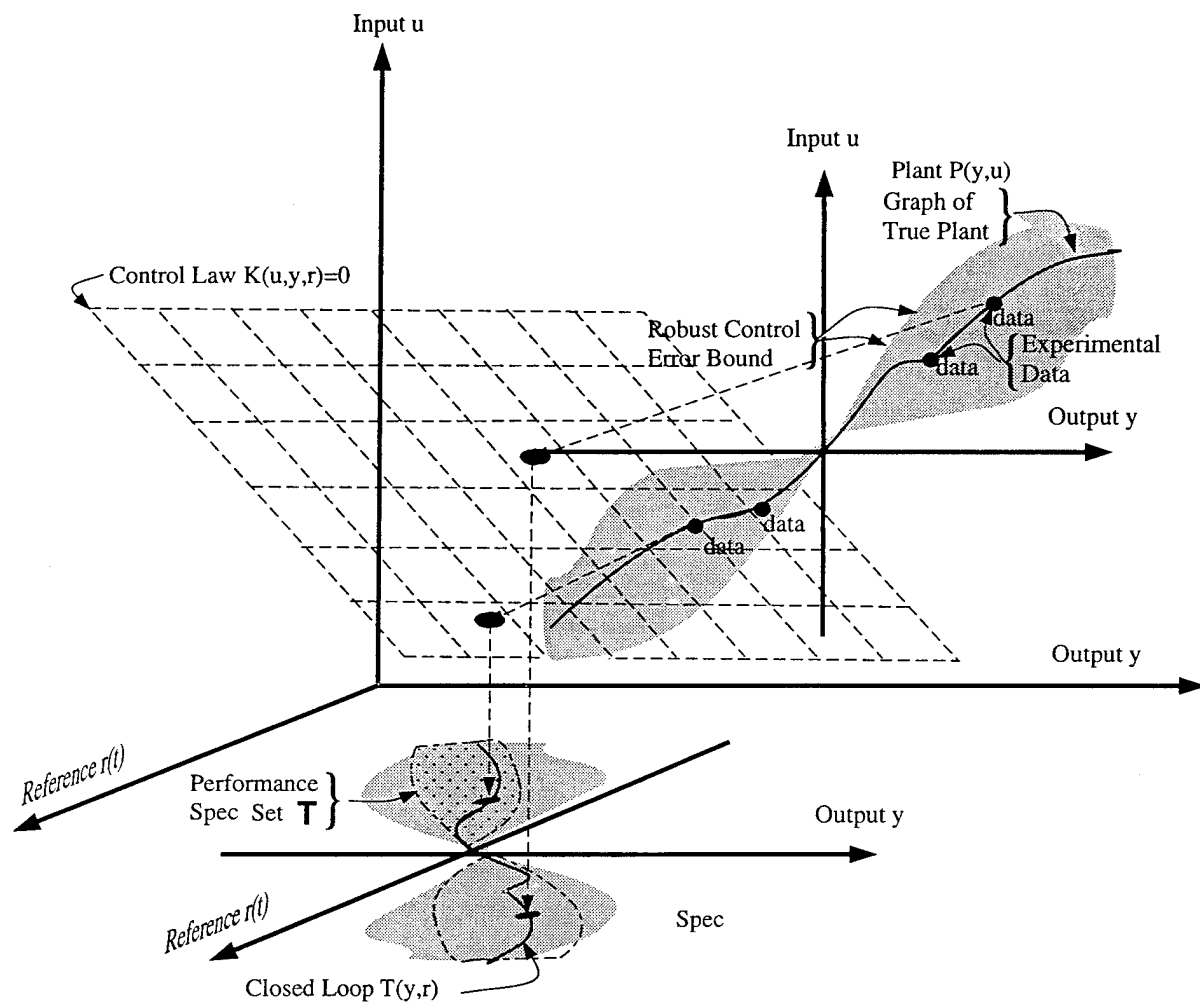


Figure 1: Controller validation from plant data (u, y) . A control law K is valid (i.e., unfalsified) if the projection under K of the data point (u, y) onto the (r, y) -plane produces a point (r, y) in the performance specification set \mathbf{T} .

We believe that this research has the potential to revolutionize the way theorists think about control, providing a much clearer understanding of the fundamental nature of learning and adaptation. On a higher plane, it would be our hope that the results will help us to begin the important task of building a solid common foundation for robust, adaptive and intelligent systems — a foundation sufficiently broad to be embraced not only by control theorists but by the artificial intelligence community as well.

3 Conclusions

With support from AFOSR Grant F49620-92-J-0014, significant progress has been made in the theory for reliable computation of H_∞ controllers, model order reduction theory, and the theory of identification of models to be used for robust control purposes, the promising new field of robust BMI synthesis theory was brought into focus and a revolutionary new unfalsified control theory was developed to aid in the understanding and design of adaptive/learning control systems. The BMI theory and the unfalsified control theory are major conceptual breakthroughs.

The *Bilinear Matrix Inequality (BMI)* approach to real/complex K_m -synthesis was demonstrated to offer enormously greater flexibility in formulating within a simple framework broad classes of robust control problems involving nonlinearities, gain-scheduling, controller order constraints, decentralized control and more. The BMI problem formulation itself is a major conceptual breakthrough because it distills the mathematical essence of robust control problems, embedding them within the conceptually simple framework of bilinear matrix inequalities. Reliable albeit sub-optimal algorithms for solving BMI's were developed, thus establishing that the BMI is more than just a superior conceptual framework since globally optimal BMI solutions can always be computed, albeit not in polynomial time since the BMI problem is in general NP-hard.

The *unfalsified control theory* developed with AFOSR support gives sharp mathematical representation of the role of experimental data in identifying robust control laws and provides a practical technique for identifying robust controllers in real-time with little or no a priori information. The theory paves the way for important links between robust and adaptive control and, perhaps, artificial intelligence. It is a conceptual breakthrough because it distills the mathematical essence of control-oriented learning in a deterministic setting by focusing sharply on what is, and is not, knowable and challenging the need for the largely gratuitous assumptions that have been the hallmark of adaptive control theories.

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Interactions (Coupling Activities)

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